FEATURES OF TURBULENT TRANSPORT OF MOMENTUM AND HEAT IN STABLY STRATIFIED BOUNDARY LAYERS AND THEIR REPRODUCTION IN ATMOSPHERIC MESOSCALE MODELS

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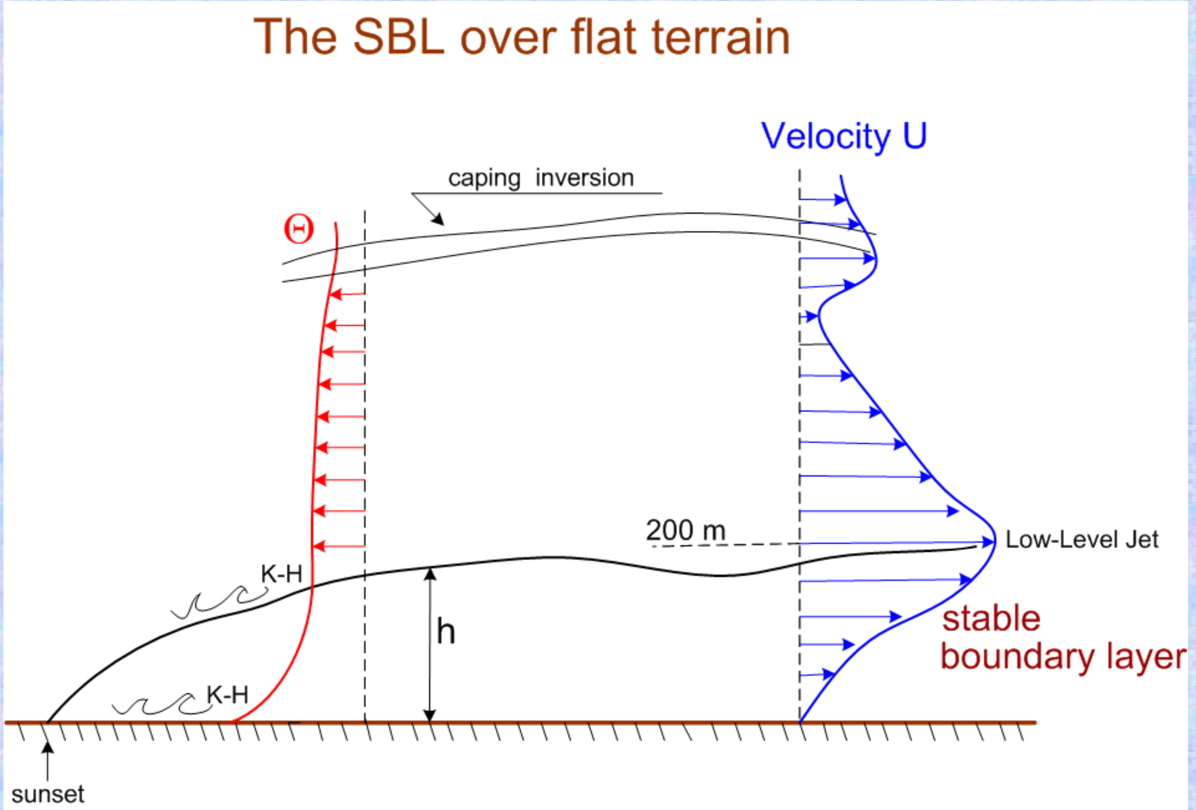
Introduction

- Boundary layers becomes stably stratified whenever surface is colder than the air. Under this condition, turbulence is generated by shear and destroyed by negative buoyancy and viscosity.

- Because of this competition between shear and buoyancy effects, the strength of turbulence in the stable boundary layer is much weaker in comparison to the neutral and convective boundary layers. As result, the stable boundary layer is also much shallower and characterized by smaller eddy motions.

Study Motivation:

▶Stable boundary layer turbulence has not received much attention despite its scientifically intriguing nature and practical significance (e.g., pollutant transport). This might be attributed to lack of adequate field or laboratory measurements and complexity of its dynamics: e.g., occurrence of Kelvin-Helmholtz instability (K-H), gravity waves, low-level jet (LLJ), etc.



Some features of the vertical turbulent transfer of momentum and scalar in stably stratified flows

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Richardson number

The relative importance of the differently directed effects of shear and stratification is characterized usually by the gradient Richardson number

$$K_m S^2 - K_h N^2 = \varepsilon \quad \Rightarrow \quad \mathbf{Ri_g} = \mathbf{N^2 / S^2}$$

$N^2 = -g(\partial \rho / \partial z) / \rho_0$ is the Brunt – Väisälä frequency

$S = \partial U / \partial z$ - is the vertical shear, of the U(z)

The gradient Richardson number is a measure of the relative intensity of density gradient in the stably stratified flows.

Being based on this criterion, in early studies it was assumed that turbulence completely attenuates, if Richardson number exceeds a certain critical value,

0,25 < $\mathbf{Ri_{gc}}$ < 1 (Richardson, 1920).

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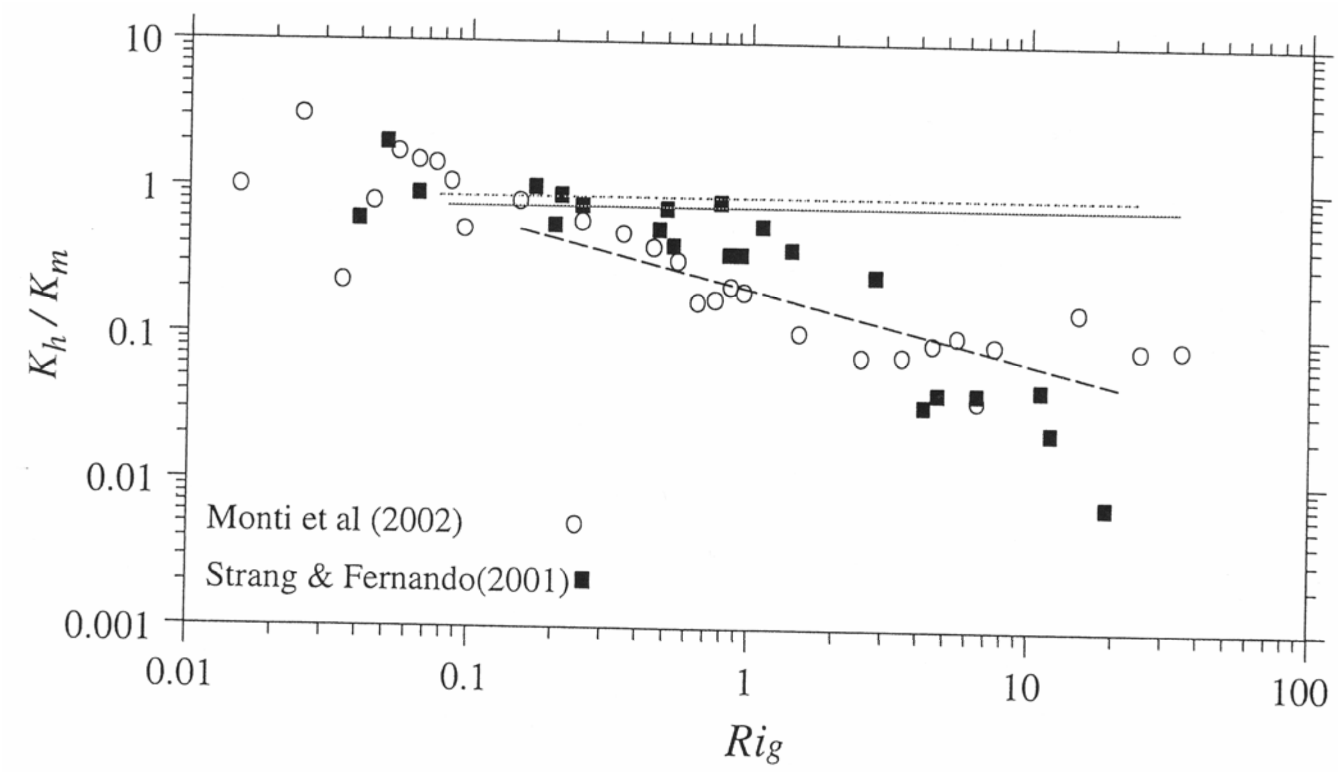
Flux Richardson number, $\mathbf{Ri_f}$

$$K_m S^2 - K_h N^2 = \varepsilon \qquad K_m S^2 (1 - Ri_f) = \varepsilon$$

$$Ri_f = Ri_g / Pr_T; \qquad Pr_T = \frac{K_m}{K_h}$$

The undamped turbulence is possible only at $\mathbf{Ri_f < Ri_{fc}}$, **and** $\mathbf{Ri_{fc}} < 1$

Inverse Prandtl number, $Pr_t^{-1} = K_H / K_M$, is stability (Ri_g) dependent



Inverse Prandtl number, $Pr_t^{-1} = K_H / K_M$ is stability (Ri_g) dependent

Wind tunnel experiment:

Ohya, Y. *Boundary-Layer Meteorology*. 2001. Vol.98, 57-82

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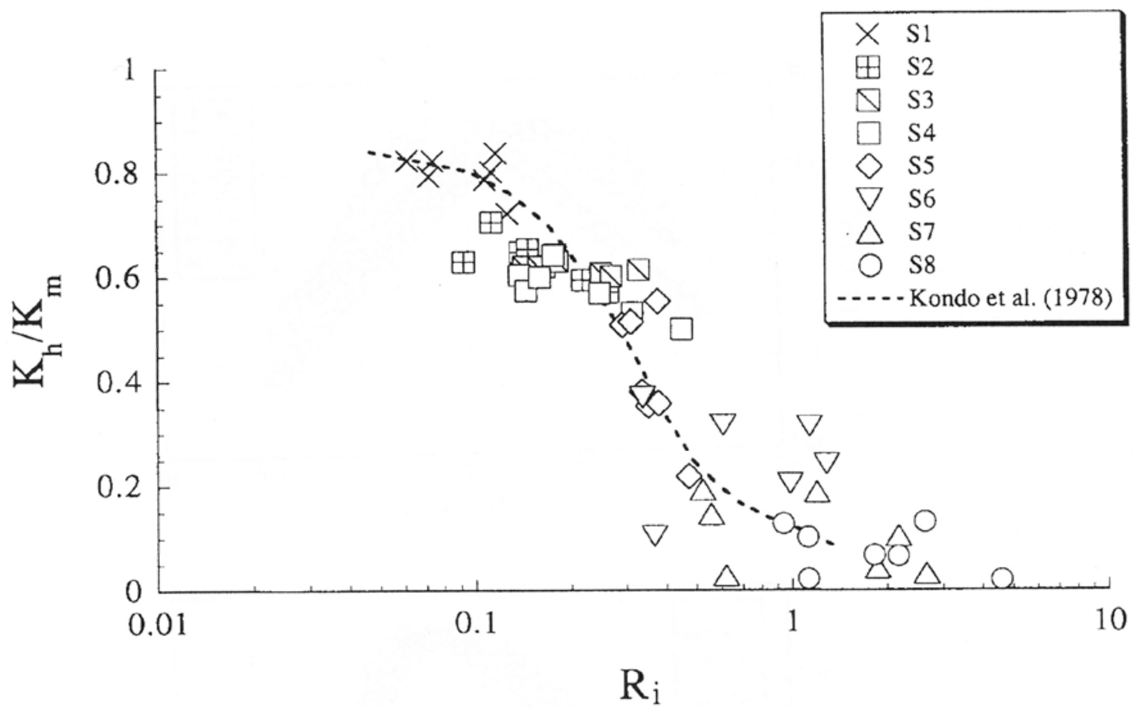
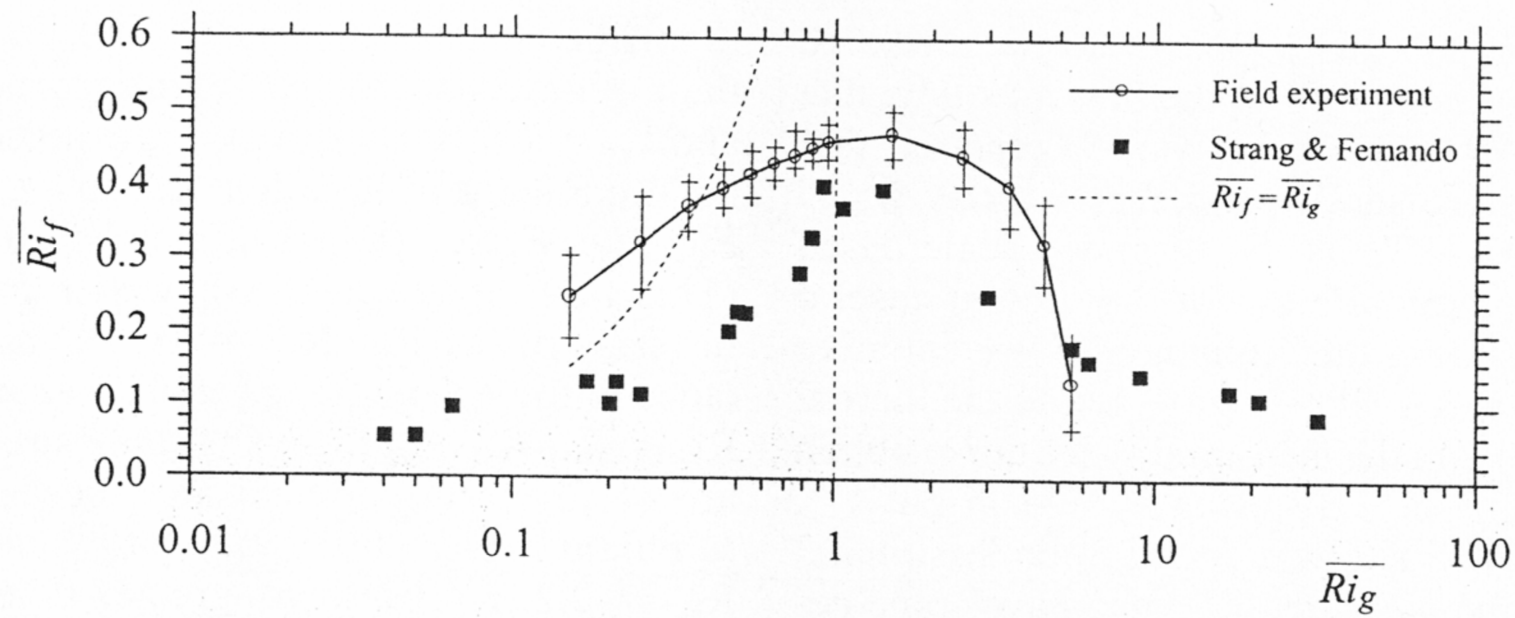


Figure 7. Correlation of the ratio of heat and momentum eddy diffusivities K_h/K_m with the local gradient Richardson number Ri.

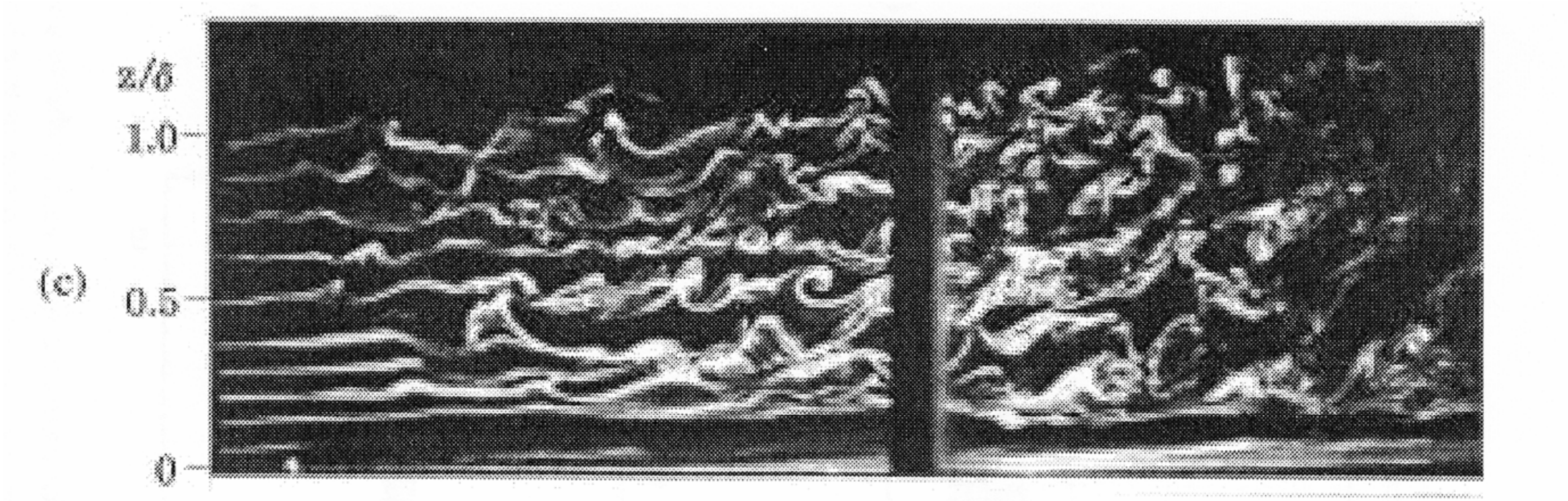
Flux Richardson number: dataset from Monti et al. (2002)



$$Ri_f = Ri_g / Pr_T$$

$$Pr_t = K_m / K_h$$

Vertical turbulent transfer, K-H instabilities and internal waves: the laboratory experiment of Ohya (2001)



Visualization by smoke shows the instantaneous picture of flow (from left to right) in the strongly steady boundary layer. Turbulent flow is formed with buoyancy and are included wave-like motions. Local shear causes instability analogous to the wave breakdown of Kelvin - Helmholtz. Generated turbulence intermixes momentum and heat, which decreases the shear and leads to an increase in Richardson number.

RANS-approach for turbulent stratified flows

$$\frac{DU_i}{Dt} = -\frac{\partial}{\partial x_j}\tau_{ij} - g_i - \frac{1}{\rho}\frac{\partial P}{\partial x_i} - 2\varepsilon_{ijk}\Omega_j U_k;$$

$$\frac{D\Theta}{Dt} = -\frac{\partial}{\partial x_j} h_j,$$

.

$\tau_{ij} \equiv \langle u_i u_j \rangle \rightarrow$ the turbulent stresses tensor

$h_i \equiv \langle u_i \theta \rangle \rightarrow$ the turbulent heat flux vector

Turbulence equations

- Reynolds stresses, $\tau_{ij} \equiv \langle u_i u_j \rangle$

$$\frac{D}{Dt}\tau_{ij} + D_{ij} = P_{ij} + \beta_i h_j + \beta_j h_i - \Pi_{ij} - \varepsilon_{ij}$$

$$P_{ij} = -\left(\tau_{ik} \frac{\partial U_j}{\partial x_k} + \tau_{jk} \frac{\partial U_i}{\partial x_k} \right) \qquad \Pi_{ij} = \langle u_i \frac{\partial p}{\partial x_j} \rangle + \langle u_j \frac{\partial p}{\partial x_i} \rangle - \frac{2}{3}\delta_{ij} \frac{\partial}{\partial x_k} \langle p u_k \rangle$$

$$D_{ij} \equiv \frac{\partial}{\partial x_k}\left(\langle u_i u_j u_k \rangle + \frac{2}{3}\delta_{ij} \langle p u_k \rangle \right) \qquad \varepsilon_{ij} = 2\nu \langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \rangle = \frac{2}{3}\delta_{ij}\varepsilon \qquad \beta_i \equiv \beta g_i$$

- Heat fluxes, $h_i \equiv \langle u_i \theta \rangle$

$$\frac{Dh_i}{Dt} + D_i^h = -h_j \frac{\partial U_i}{\partial x_j} - \tau_{ij} \frac{\partial \Theta}{\partial x_j} + \beta_i \langle \theta^2 \rangle - \Pi_{i\theta},$$

$$\Pi_{i\theta} \equiv \langle \theta \frac{\partial p}{\partial x_i} \rangle, \qquad D_i^h = \frac{\partial}{\partial x_j} \langle u_i u_j \theta \rangle$$

New dependence for the pressure correlation $\Pi_i^\theta = \overline{\theta p_{,i}}$ in the stably stratified turbulence

Relaxation linear model for the slow term: $\Pi_i^\theta = \overline{\theta p_{,i}}$ ” $\dfrac{\overline{u_i\theta}}{\tau_{p\theta}}$

‘Standard’ the SOC models usually assume, that

$$\tau_{p\theta} \text{ ” } \tau = \frac{2E}{\varepsilon}$$

<u>Such closure may not necessarily apply to the stably stratified flows!</u>
Because we use the original theoretical work of Weinstock (1989), pointed out that the time scale $\tau_{p\theta}$ must include a buoyancy damping factor

$$\tau_{p\theta} = \frac{\tau}{1 + a\tau^2 N^2}$$

‘Weinstock’s damping factor’

Full Explicit Algebraic Models for Reynolds Stresses and Scalar Fluxes

$$\frac{Db_{ij}}{Dt} + D_{ij} = 0 = -\frac{4}{3}ES_{ij} - \left(\Sigma_{ij} + Z_{ij}\right) + B_{ij} - \Pi_{ij}$$

$$\frac{Dh_i}{Dt} + D_i^h = 0 = -h_j\frac{\partial U_i}{\partial x_j} - \tau_{ij}\frac{\partial \Theta}{\partial x_j} + \beta_i\langle\theta^2\rangle - \Pi_{i\theta},$$

Algebraic equations for $b_{ij} = \langle u_i u_j\rangle / E - 2/3 \cdot \delta_{ij}$ **and** $h_i \equiv \langle u_i \theta\rangle$:

$$b_{ij} = -\alpha_1 E\tau S_{ij} - \alpha_2\left(\Sigma_{ij} + Z_{ij}\right) + \alpha_3 B_{ij}$$

$$A_{ij}h_j = -\tau\left(b_{ij} + \frac{2}{3}\delta_{ij}E\right)\frac{\partial \Theta}{\partial x_j} + \alpha_4\tau\beta g\delta_{i3}\langle\theta^2\rangle$$

Improved Full Explicit Algebraic Models for Reynolds Stresses and Scalar Fluxes : 2D case

$$\left(\langle uw\rangle, \langle vw\rangle\right) = -K_M \left(\frac{\partial U}{\partial z}, \frac{\partial V}{\partial z}\right)$$

$$K_M = E\tau S_M$$

$$K_H = E\tau S_H$$

$$\tau = \frac{E}{\varepsilon}$$

$$\langle w\theta\rangle = -K_H \frac{\partial \Theta}{\partial z} + \gamma_c$$

$$\gamma_c = \frac{1}{D}\left\{1 + \frac{2}{3}\alpha_2^2 G_M + s_6 G_H\right\}\alpha_5(\tau\beta g)\langle\theta^2\rangle$$

$$G_H \equiv (\tau N)^2 \qquad G_M \equiv (\tau S)^2 \qquad N^2 = \beta g \frac{\partial \Theta}{\partial z} \qquad S^2 \equiv \left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2$$

$$S_M = \frac{1}{D}\left\{\begin{array}{l} s_0\left[1 + s_1 G_H\left(s_2 - s_3 G_H\right)\right] + s_4 s_5 \times \\ \times\left(1 + s_6 G_H\right)\tau\beta g\left(\langle\theta^2\rangle / E\right)\end{array}\right\}$$

$$S_H = \frac{1}{D}\left\{\frac{2}{3}\frac{1}{c_{1\theta}^*}\left(1 + s_6 G_H\right)\right\}$$

$$D = 1 + d_1 G_M + d_2 G_H + d_3 G_M G_H + d_4 G_H^2 + (d_5 G_H^2 - d_6 G_M G_H) G_H$$

Three-parametric turbulence model

Turbulent kinetic energy $E = (1/2)\langle u_i u_i \rangle$

$$\frac{DE}{Dt} + \frac{1}{2} D_{ii} = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta_i h_i - \varepsilon,$$

TKE dissipation, ε

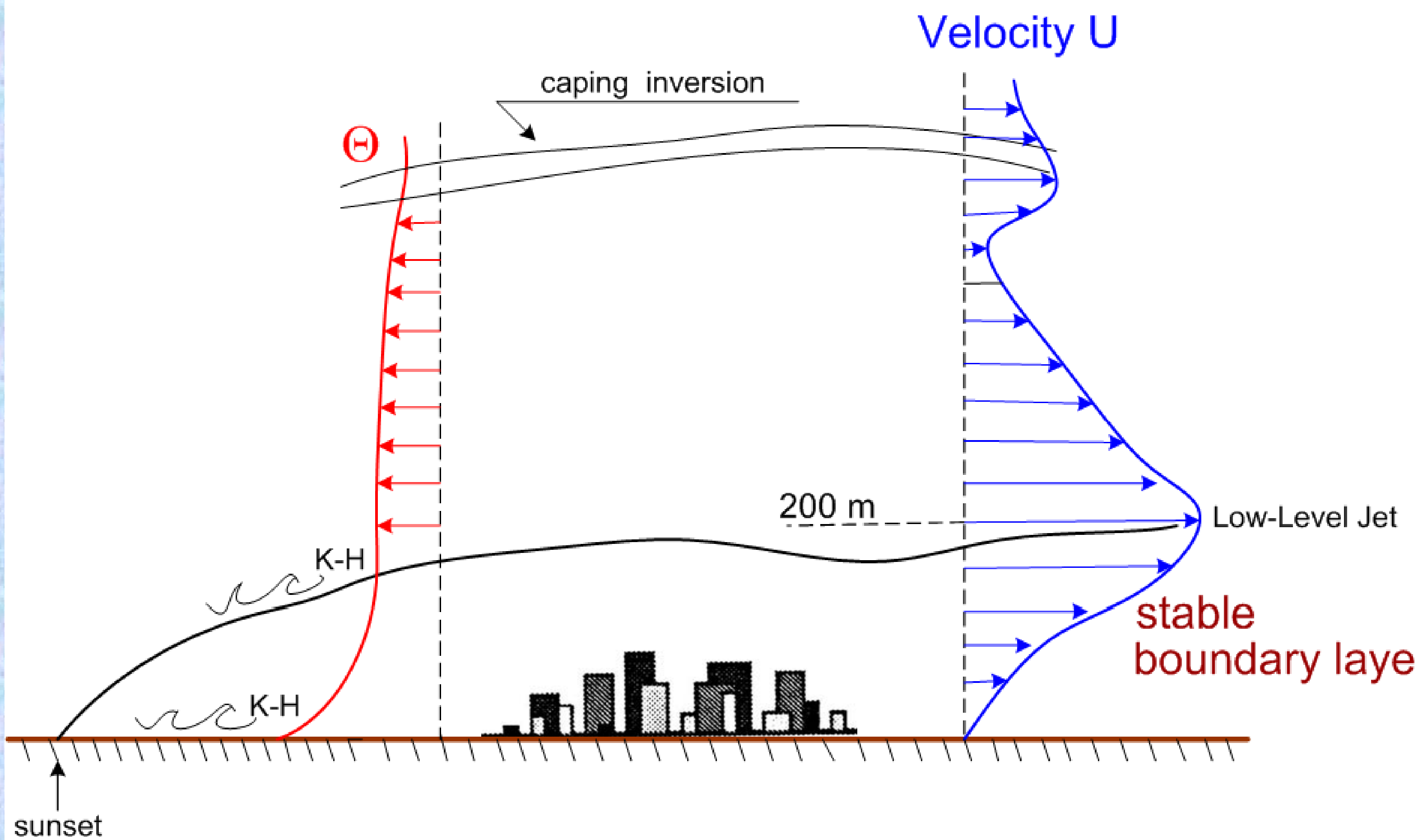
$$\frac{D\varepsilon}{Dt} + D_\varepsilon = c_{\varepsilon 1} \frac{\varepsilon}{E} \left(-\langle u_i u_k \rangle \frac{\partial U_i}{\partial x_k} + \beta g \delta_{i3} \langle u_i \theta \rangle \right) - c_{\varepsilon 2} \frac{\varepsilon^2}{E},$$

Temperature variance, $\langle \theta^2 \rangle$

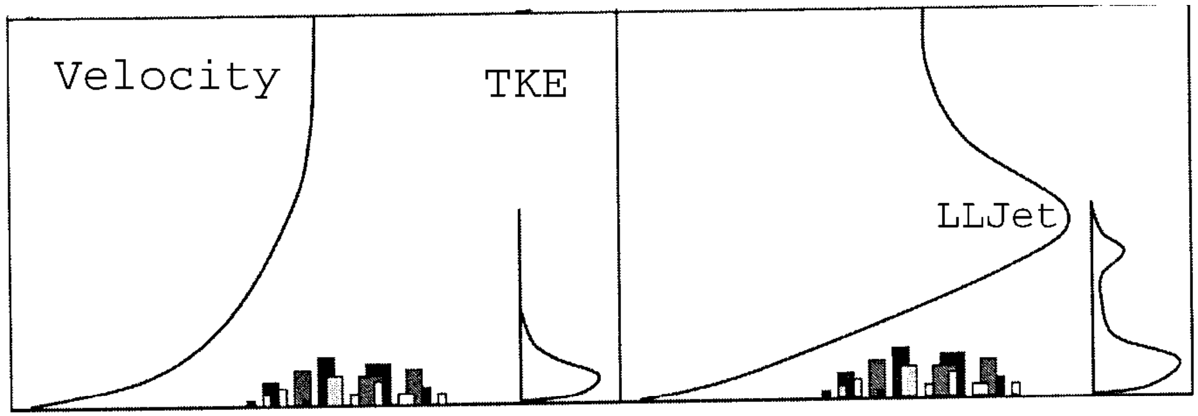
$$\frac{D\langle \theta^2 \rangle}{Dt} + D_{\theta^2} = -2h_i \frac{\partial \Theta}{\partial x_i} - 2\varepsilon_\theta,$$

Modeling and Simulation of SBL

The SBL over flat terrain

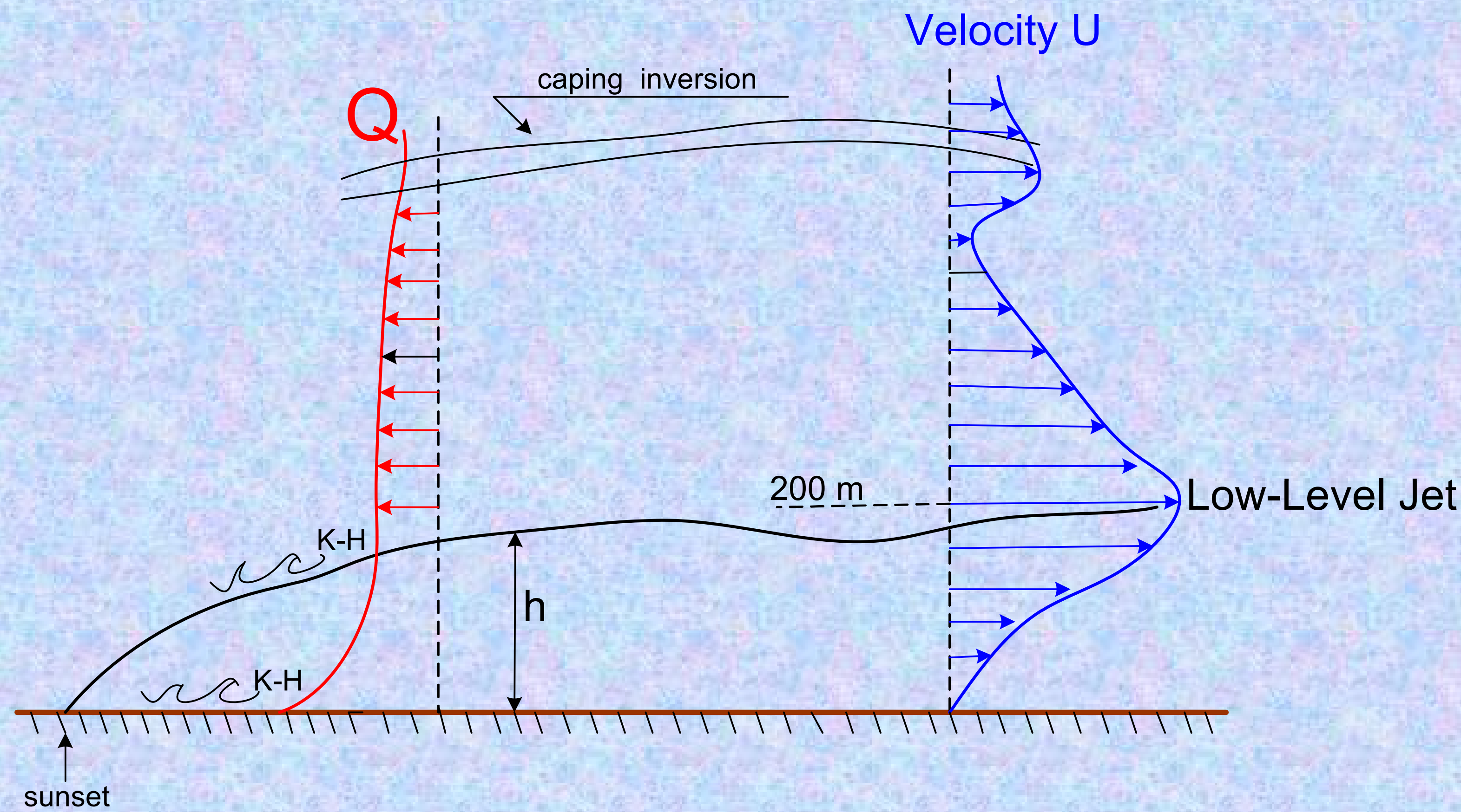


Low-Level Jet in the SBL



When the wind shear is dominant during night, the nocturnal Low-Level Jet (sketch on the right side) forms following the attenuation of convective turbulent stresses from their afternoon maximum, allowing nighttime winds above a stable boundary layer to accelerate with formation of jet maximum or "nose".

The SBL over flat terrain



Stable Boundary-Layer Experiment

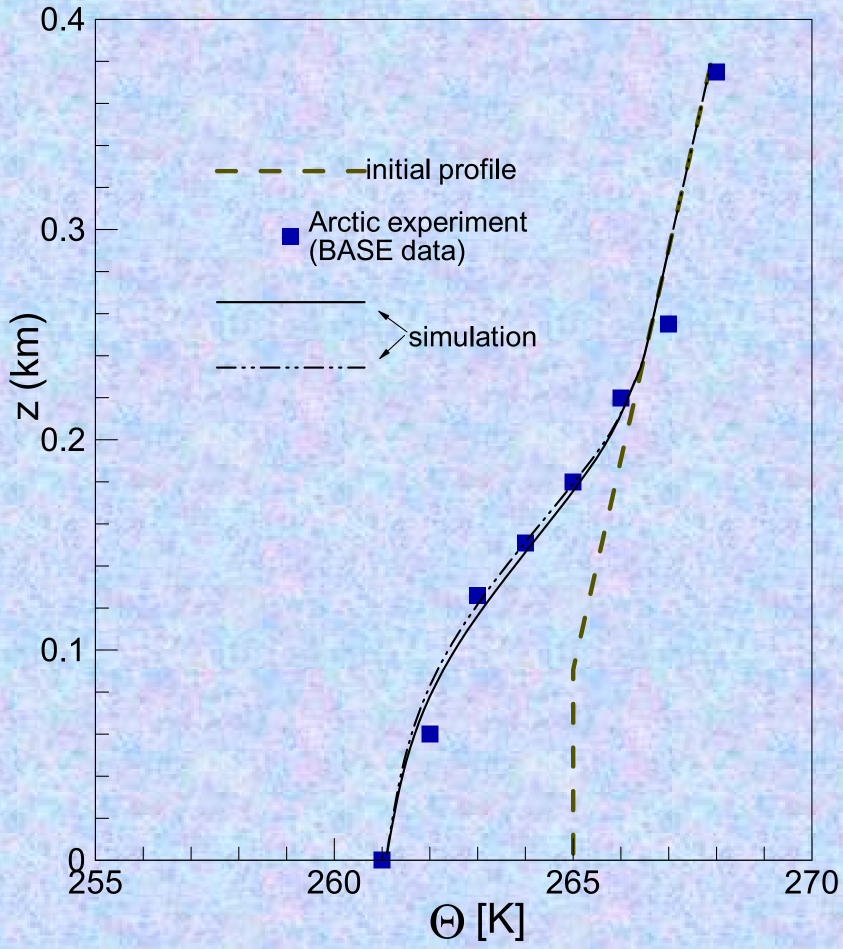
- The initial temperature profile consists of an adiabatic layer with potential temperature 265 K from surface up to 100m, above which the air is stable with a constant lapse of 0.01 K/m (it is based upon the Kosovic and Carry (2000) stable nocturnal boundary layer LES-simulations).

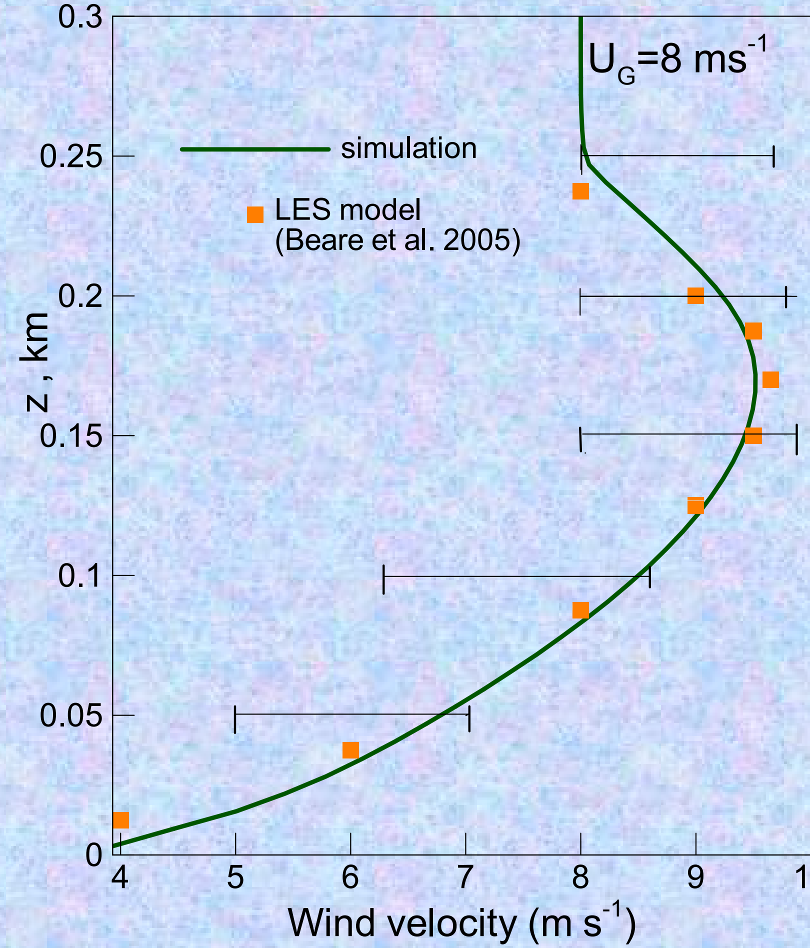
- A prescribed cooling rate of 0.5 K /hr is enforced at the surface.

- Wind profiles are initially set equal to the geostrophic wind value (8 m/s in the x-direction) throughout the layer.

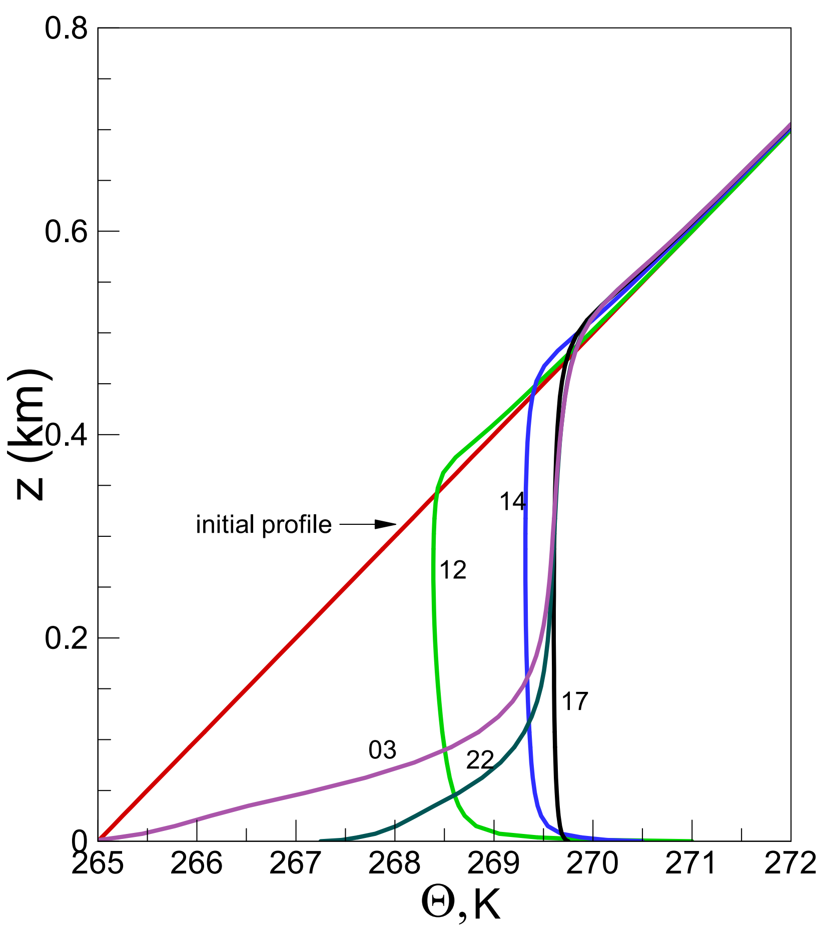
- At the ground turbulent fluxes are computing using the MOST according to the non-iterative formulation.

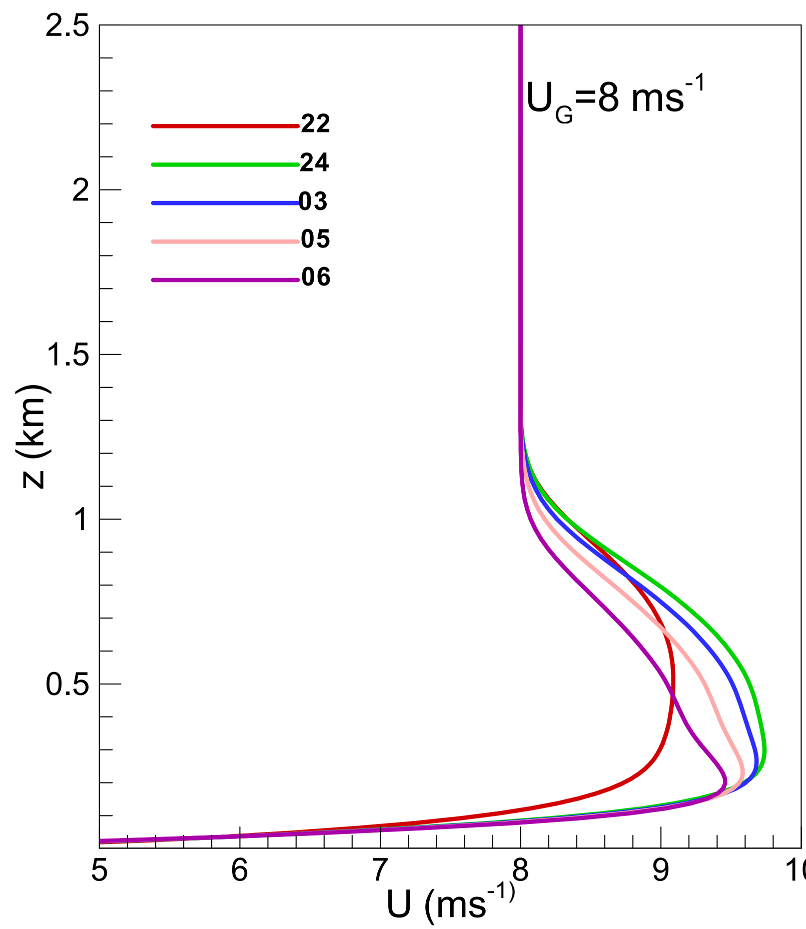
Temperature and Velocity Profiles in SBL with Low-Level Jet



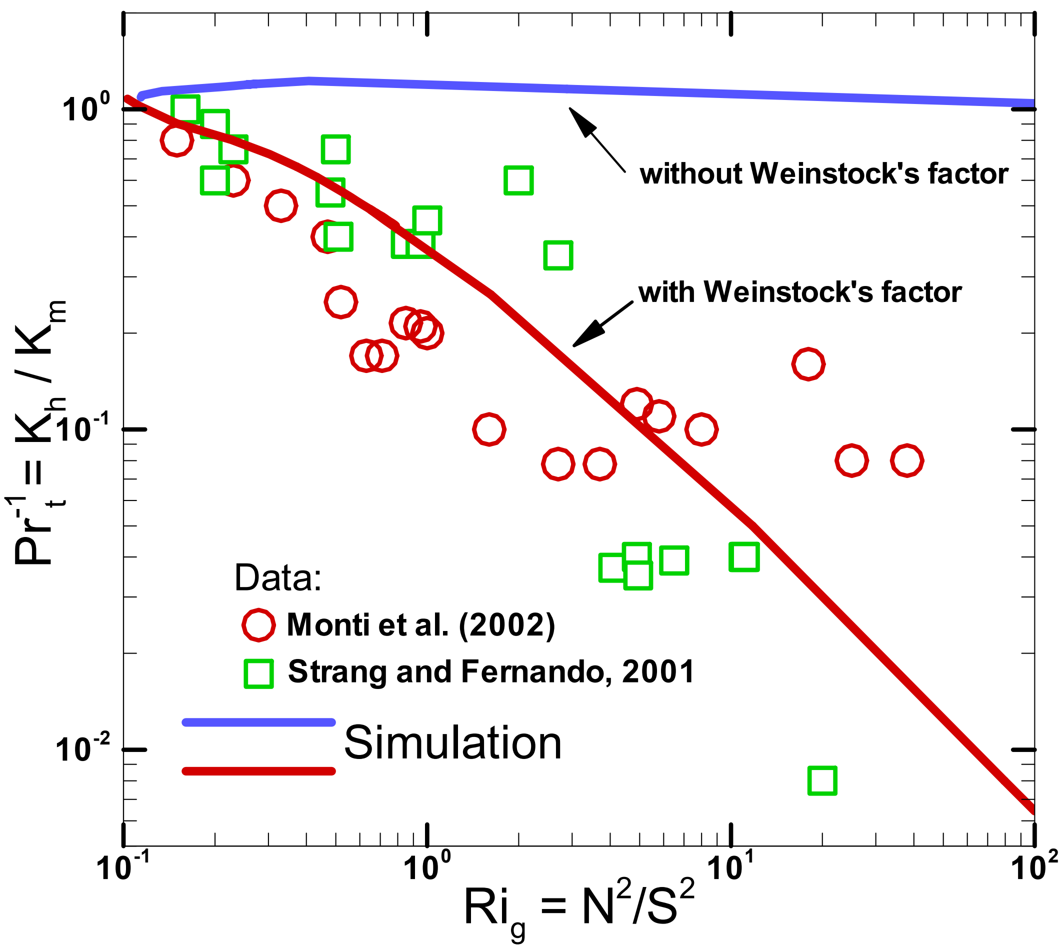


Generation of Low-Level Jet in the nocturnal ABL





Inverse Turbulent Prandtl Number, Pr_t^{-1}



Computing results of Pr_t^{-1} in the SBL over a flat terrain:

$$\tau_{p\theta} \approx \tau = E / \varepsilon$$

(without the Weinstock's factor)

$$\tau_{p\theta} = \frac{\tau}{1 + a\tau^2 N^2}$$

(with the Weinstock's factor)

$$N^2 = \beta g \cdot \partial \Theta / \partial z$$

is the Brunt-Väisälä frequency

Asymptotic: $Ri_g \to 0, \; Pr_t = K_m / K_h \to 0.88$

CONCLUSION

- The improved nonlocal turbulence model for describing the SBL over the aerodynamically rough surface with thermal inhomogeneities was presented.
- By using of simple 2D computational test the some features of turbulent transport of momentum and heat in the SBL was reproduced.
- **Simulations based on the improved expressions for turbulent fluxes of momentum and heat showed , that turbulent Prandtl number is stability dependent from Richardson number and momentum can be transferred more effectively than heat under stably stratified conditions.**

THANK YOU!