Main factors of climate variability and their application for environment protection problems in Siberia

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Algorithms
for revealing climatic variability

- **Singular vectors (SV) for forward tangent operator of dynamical models and the use of SV-decomposition for scenario construction and errors analysis (uncertainty reducing);**
- **ensembles of prognostic scenarios with generation of perturbations (“breeding cycle”, Lyapunov’s vectors);**
- **Monte-Carlo methods for scenario construction**
- **Stochastic-dynamic moment equations and Liouville equations**

ICMMG technology

- **Orthogonal decomposition of the phase spaces of non-linear dynamical systems for formation of informative basis subspaces;**
- **Minimization of uncertainties with respect to given criteria of prognosis quality (+ data assimilation if any)**

Scenarios construction and adaptive monitoring with SV

$$\frac{\partial \varphi}{\partial \boldsymbol{t}} + \boldsymbol{A}(\varphi) = 0 \quad \Rightarrow$$

Tangent linearization about $\tilde{\varphi}(\mathbf{x}, \boldsymbol{t})$

$$\frac{\partial \delta\varphi}{\partial \boldsymbol{t}} + \boldsymbol{A}_L \delta\varphi = 0 \qquad \delta\varphi(\mathbf{x}, 0) = (\boldsymbol{a\ priori})$$

$$\delta\varphi(\mathbf{x}, \boldsymbol{t}) = \boldsymbol{L}\delta\varphi(\mathbf{x}, 0), \ \vec{\boldsymbol{x}} \in \boldsymbol{D}, \ \boldsymbol{t} \in \left[0, \overline{\boldsymbol{t}}\right]$$

$L(\mathbf{x}, t)$ - forward tangent propagator about $\tilde{\varphi}(\mathbf{x}, \boldsymbol{t})$

$$\psi(\mathbf{x}, 0)\Big|_{\boldsymbol{D}} = \boldsymbol{L}^{*}\left[\delta\varphi(\mathbf{x}, \boldsymbol{t})\right]_{\Sigma_t}, \boldsymbol{t} \in \left[\overline{\boldsymbol{t}} \to 0\right]$$

Basic relations and patterns for SVs

$$\left\|\delta\varphi(\boldsymbol{t})\right\|_{\Sigma_{\boldsymbol{t}}} = \left(\delta\varphi(\boldsymbol{t}), \delta\varphi(\boldsymbol{t})\right) =$$

$$= \left(\boldsymbol{L}\delta\varphi(0), \boldsymbol{L}\delta\varphi(0)\right) = \left(\delta\varphi(0), \boldsymbol{L}^{*}\boldsymbol{L}\delta\varphi(0)\right) =$$

$$= \left(\delta\varphi(0), \psi(0)\right)$$

$\sum_{\boldsymbol{t}} \in \boldsymbol{D}$ evaluation domain at $\boldsymbol{t} = \overline{\boldsymbol{t}}$

$\sum_{0} \in \boldsymbol{D}$ target area at t=0

$\left[0, \overline{\boldsymbol{t}}\right]$ "optimal" time interval ($\leq$ 48 h)

Partial eigenproblem for SVs

$$L^{*}LV_i = \sigma_i^2 V_i, \quad (i \in K)$$

σ_i, V_i singular values and vectors of L (SEVs, SVs)

•Lanzosh algorithm
•Ortogonal decomposition of perturbation spaces
•Optimal construction of perturbations
with respect to rapidly growing SVs

Structuring and decomposition of data bases

Initial data base $\Phi \equiv \left\{ \varphi(\vec{x}, t, \vec{Y}) \in Q(D_t) \subset R_N, \vec{Y} \in R(D_t) \right\}$

Structured data base $Z = \left\{ z_i = C^{1/2} \varphi_i,\ i = \overline{1, n},\ \varphi_i \in R_N \right\}$

Z $n \times N$ matrix of vectors from $R_n \times R_N$

C $N \times N$ diagonal matrix of total energy weight of φ

Scattering function $S(v) = (v^T Z^T Z v) = (v^T \Gamma v)$

Orthogonal decomposition of Z on the base of optimal properties of $S(v)$

$$\begin{cases} \Gamma v = \lambda v \Rightarrow \{\lambda_p, v_p \in R_n\}, \ \Psi_p \in R_N \\ v_p^T v_q = \lambda_p \delta_{pq}, \Psi_p^T \Psi_q = \delta_{pq}, \ p,q = \overline{1,n} \end{cases}$$

$$\begin{cases} V = \{v_p\} \\ \Lambda = diag\{\lambda_p > 0\} \end{cases}, \quad n \times n \text{ matrices}$$

$$\Psi \equiv \{\Psi_p \in R_N, \ p = \overline{1,n}\}, \ n \times N \text{ matrix}$$

$\Psi \equiv Z V \Lambda^{-1}$ decomposition algorithm

$\tilde{Z} = \Psi V^T$ reconstruction algorithm

Factor subspaces
for deterministic- stochastic scenarios

- Factor spaces

$$\vec{\varphi} = X_0 + \vec{\xi}$$

is a linear subset of the vector space $X \subset DATA$

$\vec{\xi}$ is arbitrary element from X

!! Algebraic operations in X leave X_0 invariant

X_0 is the leading phase space,

$\vec{\xi}$ are generated perturbations

Construction of the vector set X_0

$$X_0 = \left\{ \sum_{1}^{n_d} c_i \Psi_i ,\ n_d \le n, 0 \le c_i \le \max|\sigma| \right\}$$

Formation of vectors X

1. Deterministic case:
 calculation by means of the process models
2. Deterministic-stochastic case:

c_i generation by means of the stochastic processes of the fractal type described by gaussian process with variance

$$\sigma_q^2 = \lambda_q^{2H}, 0 \le H \le 1$$

H is a parameter of the fractal size,

λ_q are the eigenvalues of the Gram matrix

Forming the guiding phase space with allowance for observation data on the subdomain

$\mathbf{Z}^m(\mathbf{x},\tau)$ measured data; $\Psi_p(\mathbf{x},t)$ basis

$$\mathbf{Z}(\mathbf{x},t)=\sum_{p=1}^{n_a} a_p^m \Psi_p(\mathbf{x},t),\ \ (\mathbf{x},t)\in D_t,\ \ n_a\le n$$

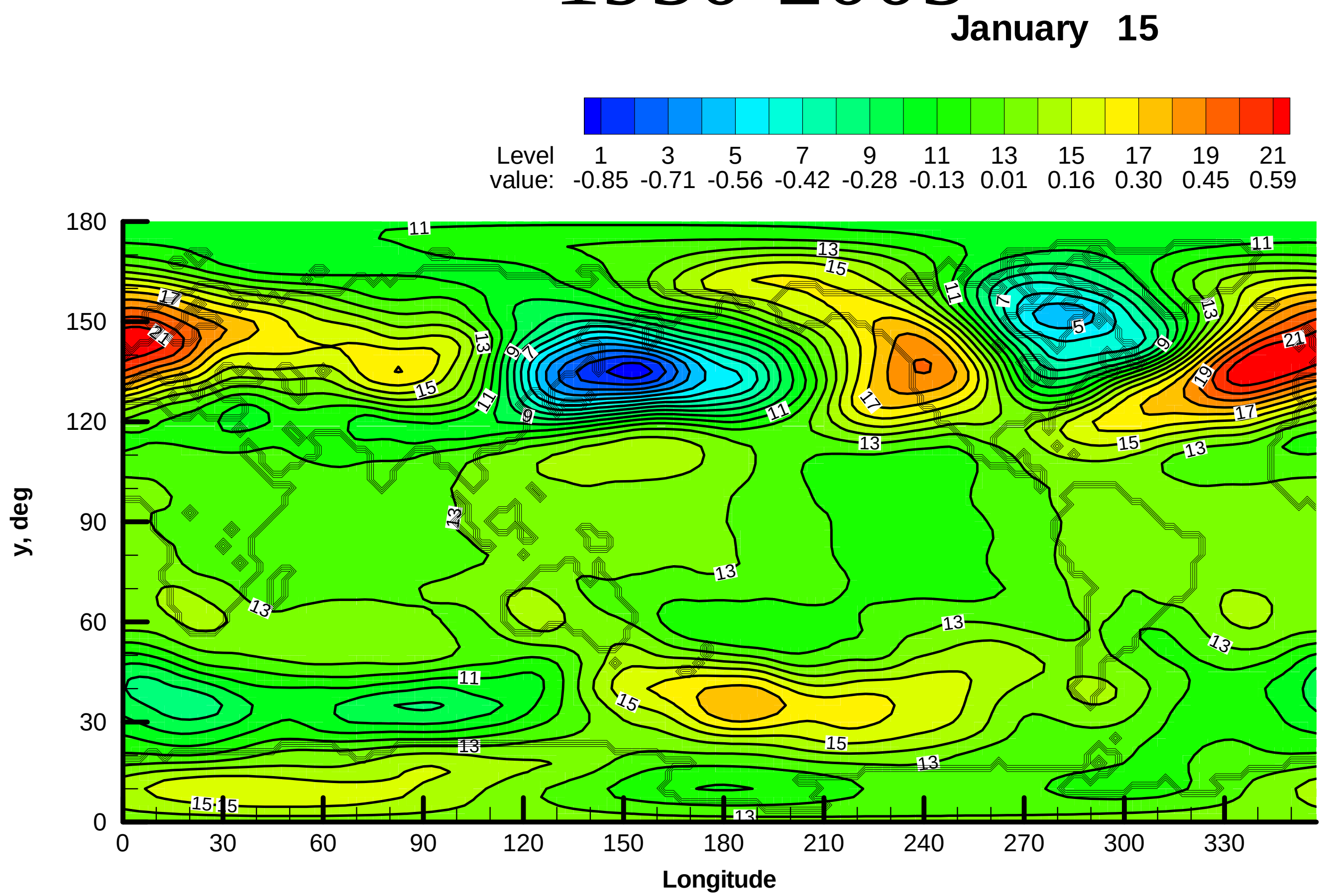
$$\min_{\langle \mathbf{a}_p^m\rangle}\left\| \mathbf{Z}^m(\mathbf{x},\tau)-\sum_{p=1}^{n_a} a_p^m \Psi_p(\mathbf{x},\tau)\right\|_{D_\tau^m}^2,\ \ (\mathbf{x},\tau)\in D_\tau^m,\ \ n_a\le n$$

$$\mathbf{a}=\left(\Gamma^m\right)^{-1}\mathbf{F}^m \qquad \mathbf{a}=\left\{a_p^m,\ \ p=\overline{1,n_a}\right\}$$

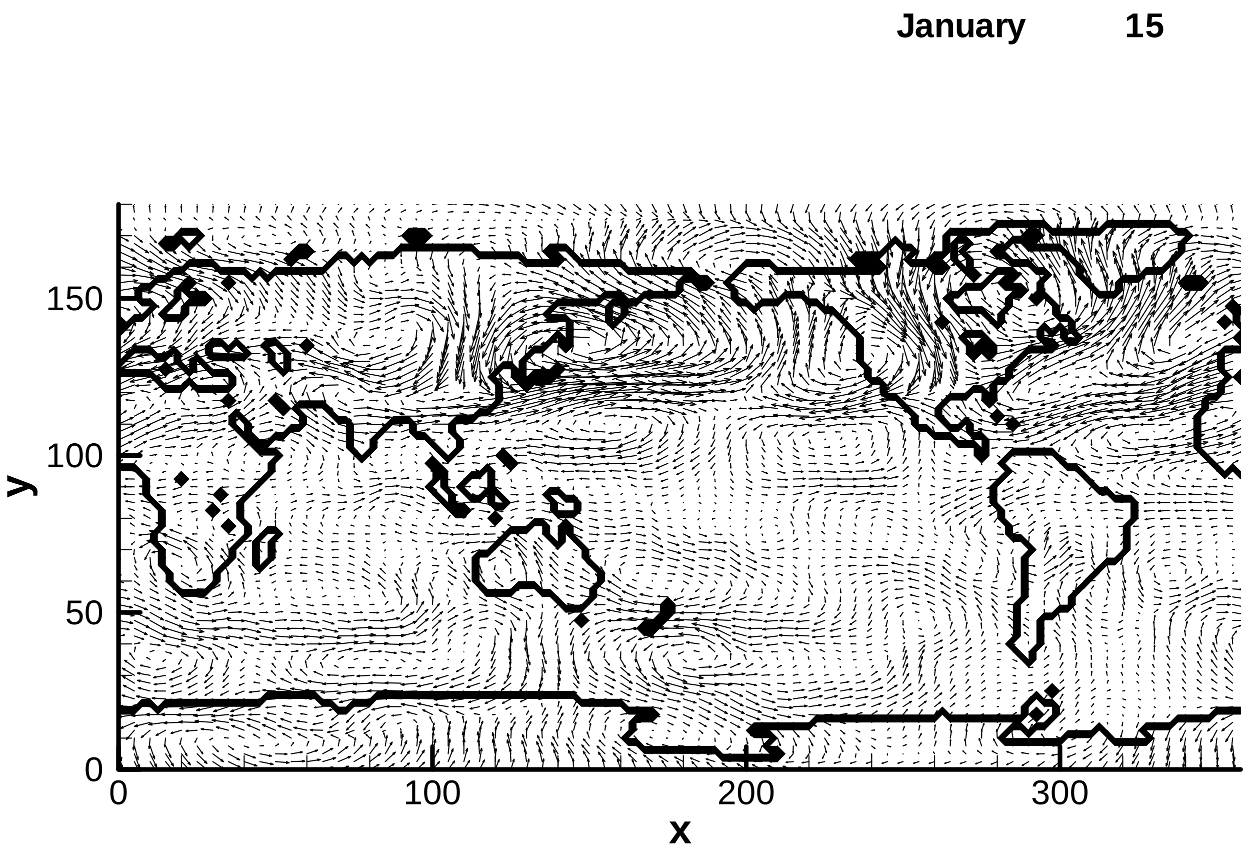
$$\Gamma^m=\left\{\Gamma_{pq}^m=\left(\Psi_p, W^m\Psi_q\right)_{D_\tau^m},\ \ p,q=\overline{1,n_a}\right\} \qquad \mathbf{F}^m=\sum_{p=1}^{n_a}\left(\Psi_p, W^m\mathbf{Z}^m\right)_{D_\tau^m}$$

If $\tau<t$ then $\mathbf{Z}(\mathbf{x},t)$ is forecast !

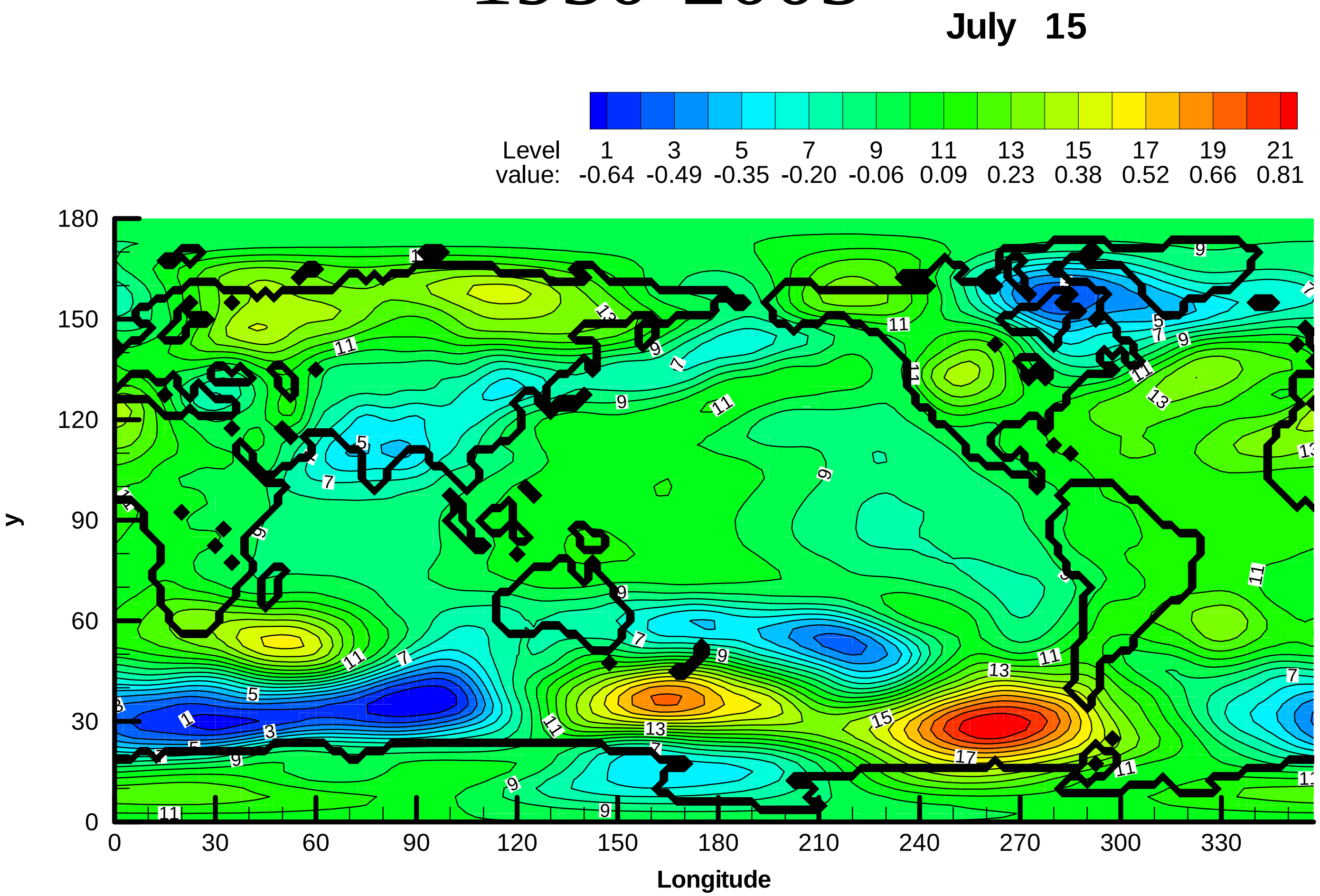
Winter pattern of the global 500-hPa geopotential height (the 1st main factor) 1950-2005



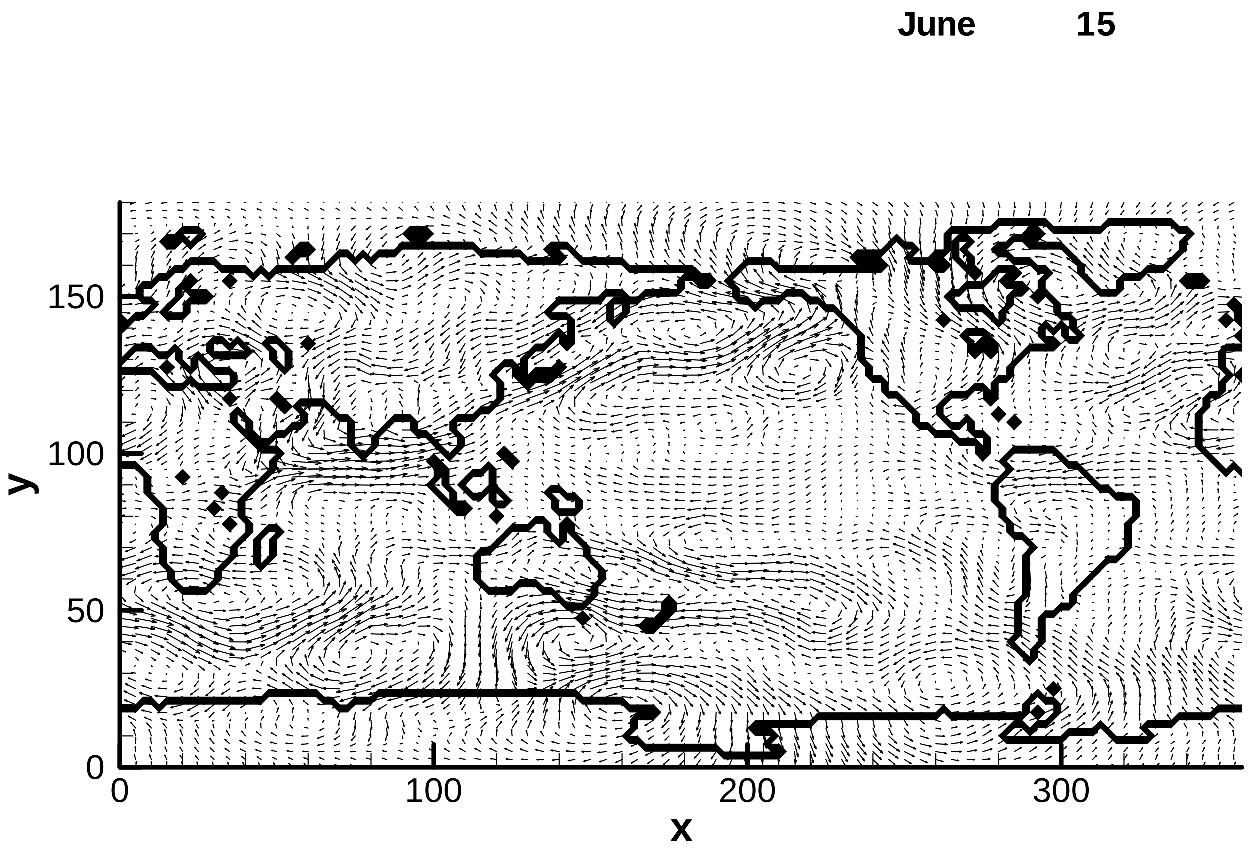
Winter pattern of global circulation (the 1st main factor) 1950-2005

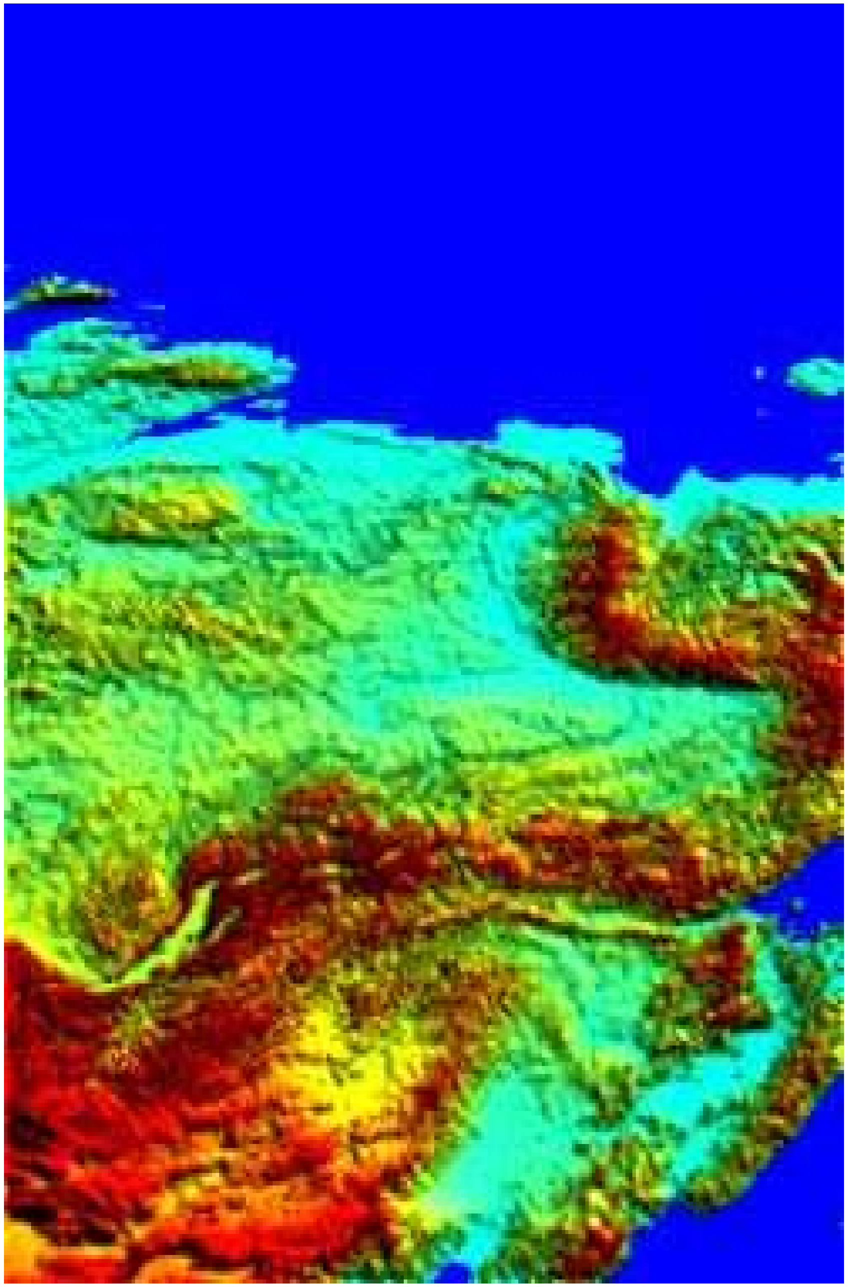


Summer pattern of the global 500-hPa geopotential height (the 1st main factor) 1950-2005



Summer pattern of global circulation (the 1st main factor) 1950-2005





East Siberia Region

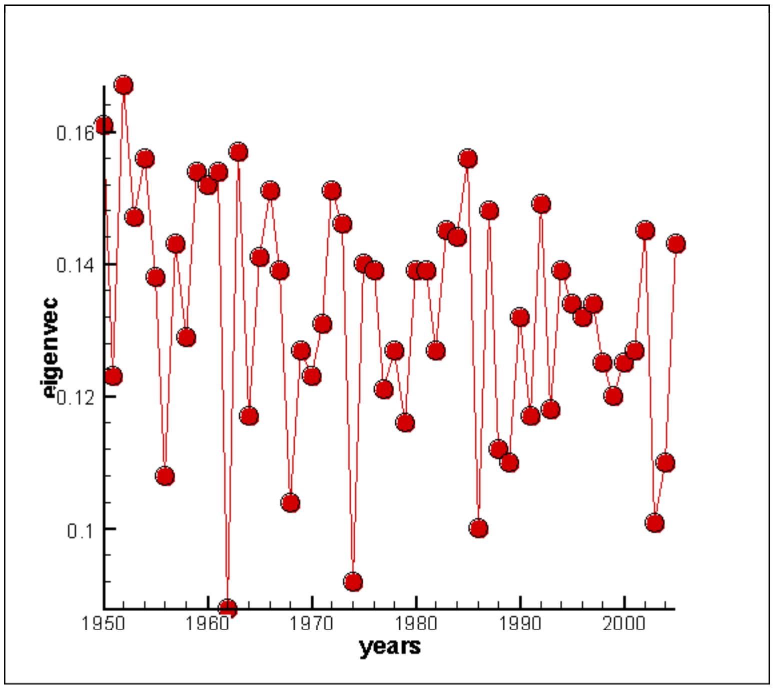
90-140 E, 45-65 N

June 1950-2005

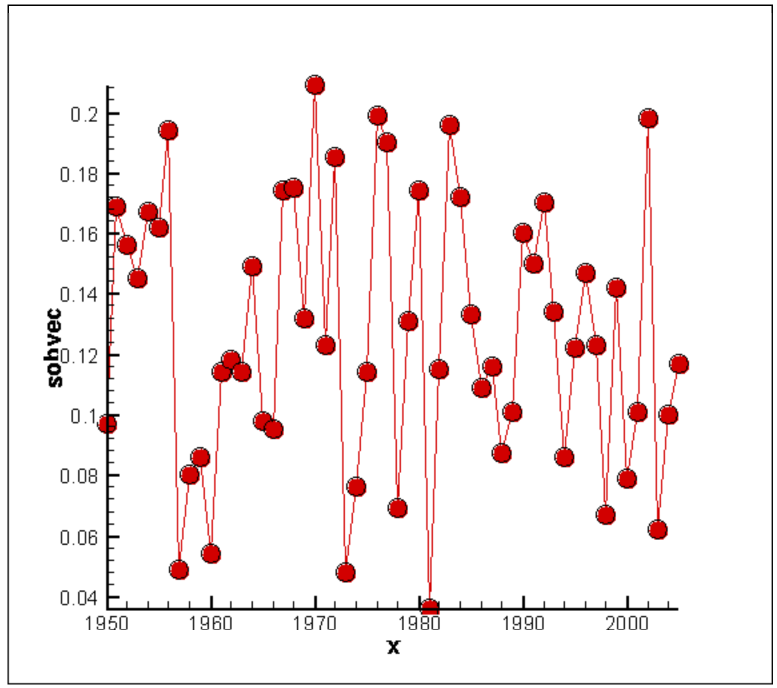
Variability of the phase spaces with respect to the first main factors

Eigenvectors N1, June, 1950-2005

Global, 17%

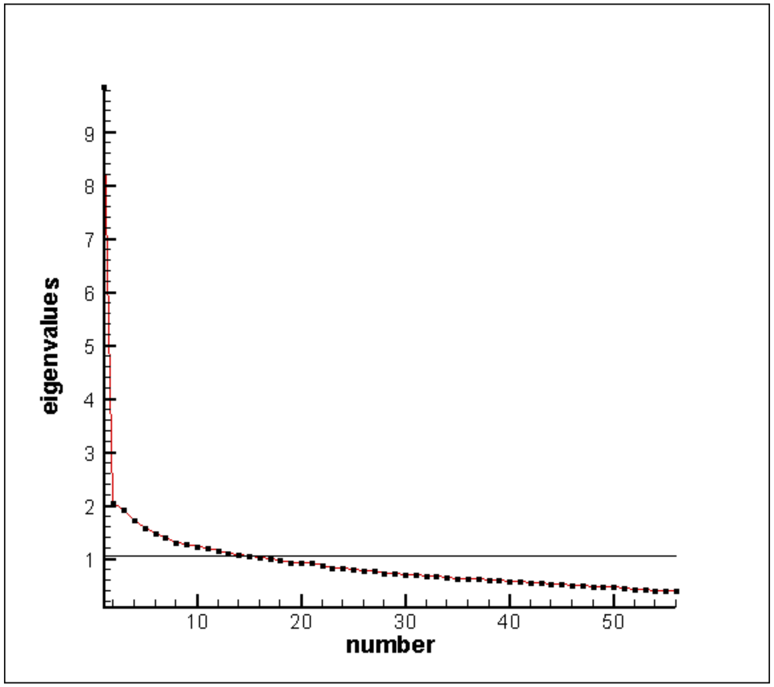


Eastern Siberia, 17,9%

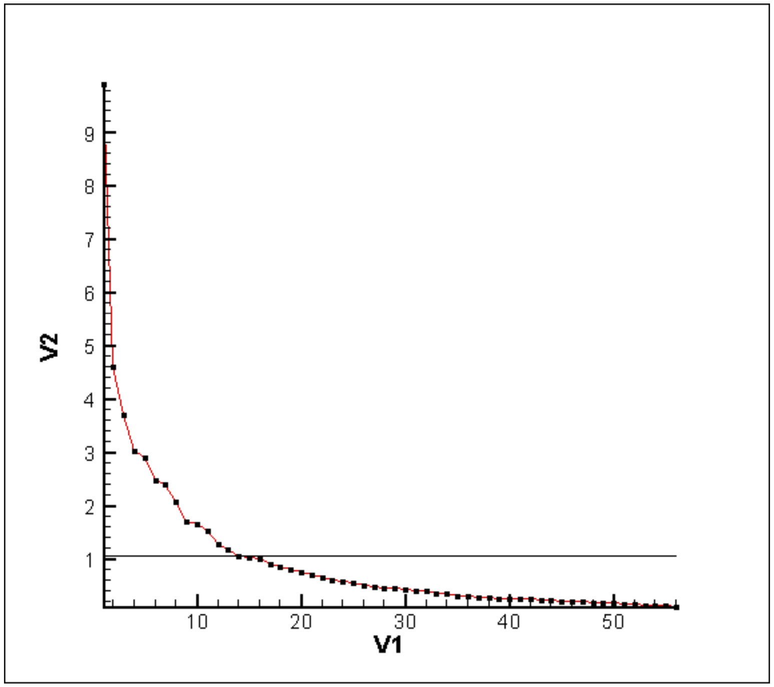


Quantification of subspace scales

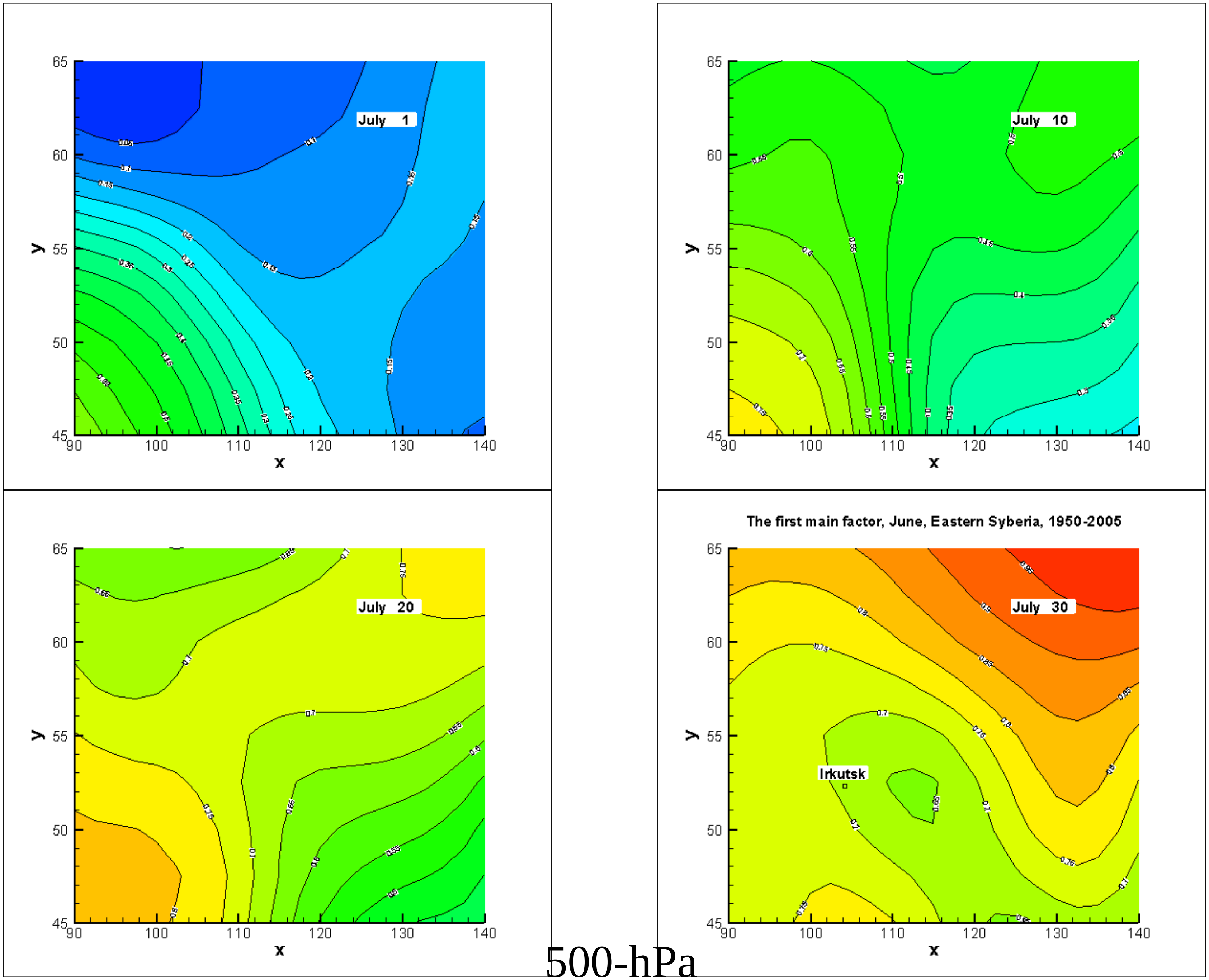
Eigenvalues of Gram matrices, June, 500-hPa, 1950-2005



Global scale

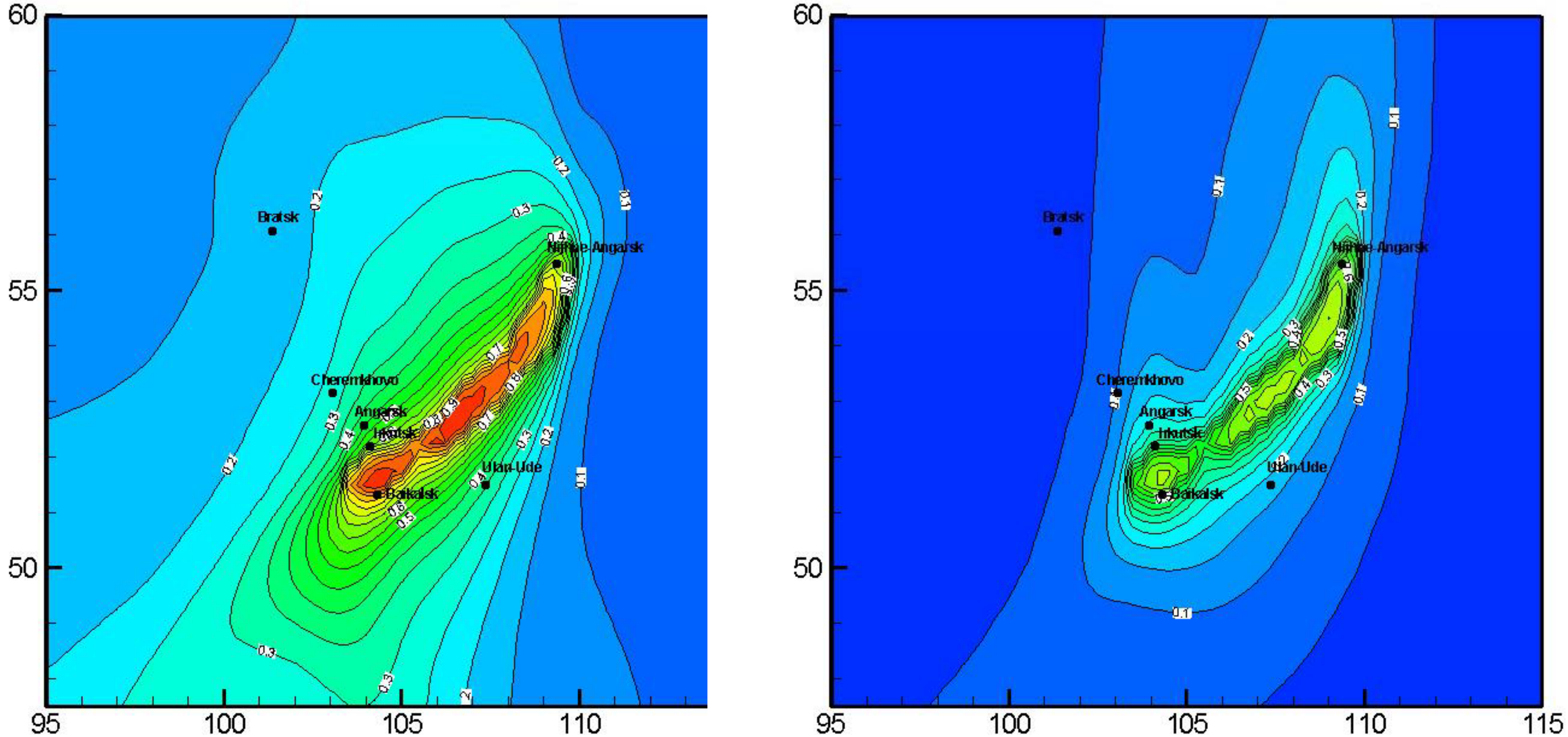


Regional scale


July 1
July 10
July 20
The first main factor, June, Eastern Syberia, 1950-2005
July 30
Irkutsk
y
x
65
60
55
50
45
90
100
110
120
130
140

500-hPa

Monthly risk functions for Lake Baikal



July

December

Conclusion

- The set of numerical algorithms for orthogonal decomposition of the phase spaces of dynamical system evolution is developed for climate and ecology studies
- The methods are applied for construction of long-term scenarios for risk assessment with respect to anthropogenic impact
- This allows us to take into account climatic data for environmental studies of global and regional scale

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